Q. Show $E^2 - c^2B^2$ is invariant under Lorentz transformation.

Ans: We have the transformation result for E and B for two inertial frame of references S and S', when S is at rest and S' is moving with respect to S with a constant translational velocity v.

The transformation rule for the diffent Component of $\vec{E}$ and $\vec{B}$ are as follows.

$$E_x = E_x' \quad , \quad E_y = \Gamma(E_y' + vB_z') \quad , \quad E_z = \Gamma(E_z' - vB_y') \quad \text{---(1)}$$

$$B_x = B_x' \quad B_y = \Gamma\left(B_y' - \frac{v}{c^2}E_z'\right) \quad B_z = \Gamma\left(B_z' + \frac{v}{c^2}E_y'\right) \quad \text{---(2)}$$

Now $E^2 - c^2B^2$

$= (E_x^2 + E_y^2 + E_z^2) - c^2(B_x^2 + B_y^2 + B_z^2)$.

using the tranferation rule from (1) and (11) and simplifying.

$$\left\{E_x'^2 + \gamma^2(E_y' + vB_z')^2 + \gamma^2(E_z' + vB_y')^2\right\}$$

$$- c^2\left[B_x'^2 + \gamma^2\left(B_y' - \frac{v}{c^2}E_z'\right)^2 + \gamma^2\left(B_z' + \frac{v}{c^2}E_y'\right)^2\right]$$

$$= E_x'^2 + \gamma^2\left\{E_y'^2 + v^2B_z'^2 + 2vE_y'B_z'\right\} + \gamma^2\left\{E_z'^2 + v^2B_y'^2 - 2vE_z'B_y'\right\}$$

$$- c^2\left[B_x'^2 + \gamma^2\left\{B_y'^2 + \frac{v^2E_z'^2}{c^4} - \frac{2B_y'E_z'}{c^2}\right\} + \gamma^2\left\{B_z'^2 + \frac{v^2E_y'^2}{c^4} + \frac{2vB_z'E_y'}{c^2}\right\}\right]$$

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Combining the components of E's and B's

$$= E_x'^2 + E_y'^2\left(\gamma^2 + \gamma^2\frac{v^2}{c^2}\right) + E_z'^2\left(\gamma^2 + \gamma^2\frac{v^2}{c^2}\right)$$

$$- c^2\left\{B_x'^2 + B_y'^2\left(\gamma^2 + \gamma^2\frac{v^2}{c^2}\right) + B_z'^2\left(\gamma^2 + \gamma^2\frac{v^2}{c^2}\right)\right\}$$

$$+ \gamma^2\left(2vE_y'B_z' + 2vE_z'B_y' + 2vB_y'E_z' - 2vE_y'B_z'\right)$$

$$= E_x'^2 + E_y'^2 + E_z'^2 - c^2\left(B_x'^2 + B_y'^2 + B_z'^2\right)$$

$$= E'^2 - c^2B'^2$$

$$\therefore E^2 - c^2B^2 = E'^2 - c^2B'^2 \quad \text{---(iii)}$$

Hence This means $(E^2 - c^2B^2)$ is invariant under Lorentz transformation

$$\left[\gamma^2 + \gamma^2\frac{v^2}{c^2}\right] = \gamma^2\left(1 + \frac{v^2}{c^2}\right)$$

$$= \frac{1 + \frac{v^2}{c^2}}{1 + \frac{v^2}{c^2}} = 1$$

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